“We heard what you said but we knew what you meant”

Automatic Reformulation in MiniZinc

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Overview

- A little bit about MiniZinc
  - Predicates, functions, and flattening
- Automatic Reformulations
  - Linearization, Sets, and Strings
  - Multi pass compilation
  - Autotabling
  - Symmetry detection
  - Globals detection
- Conclusion
“Alone we can do so little; together we can do so much.” – Helen Keller

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MiniZinc Predicates

- MiniZinc is based on model rewriting

- Predicates: define a new (global) constraint
  
  ```
  predicate alldifferent(array[int] of var int: x)
  = forall(i,j in index_set(x) where i < j)
    (x[i] != x[j]);
  ```

- Essential to treatment of globals
  - solvers use a default decomposition, or
  - replace with their own decomposition or direct constraint
    ```
    predicate alldifferent(array[int] of var int: x);
    ```

- Advantages: all globals available for all solvers
MiniZinc Functions

- It's also useful to have functions

  ```
  function array[int] of var int: global_cardinality
    (array[int] of var int: x, array[int] of int: v)
  = let { array[index_set(v)] of var int: c
      = [ sum(i in index_set(x))(x[i] = v[j])
          | j in index_set(v) ]; }
    in c;
  ```

- Common subexpression elimination is better
  - almost a third of the global constraint catalog are functions

- It also makes the MiniZinc core simpler

  ```
  function var int: abs(var int: x) =
    let { int: m = max(-lb(x),ub(x));
      var -m..m: y;
      constraint int_abs(x,y); }
    in y;
  ```
Flattening

- Mapping a high level model
  - complex loops
  - deep expressions
  - functions and predicates

- To a flat model
  - variables
  - constraints
  - objective

```plaintext
% (square) job shop scheduling in MiniZinc
int: size; % size of problem
array [1..size,1..size] of int: d; % task durations
int: total = sum(i,j in 1..size) (d[i,j]); % total duration
array [1..size,1..size] of var 0..total: s; % start times
var 0..total: end; % total end time

predicate no_overlap(var int:s1, int:d1, var int:s2, int:d2) =
  s1 + d1 <= s2 /
  s2 + d2 <= s1;

constraint
  forall(i in 1..size) ( %
    forall(j in 1..size-1) (s[i,j] + d[i,j] <= s[i,j+1]) /
    s[i,size] + d[i,size] <= end /
    forall(j,k in 1..size where j < k) ( %
      no_overlap(s[j,i], d[j,i], s[k,i], d[k,i])
    )
  );
solve minimize end;

array[0..3] of var 0..14: s;
var 0..14: end;
var bool: b1;
var bool: b2;
var bool: b3;
var bool: b4;
constraint int_lin_le ([1,-1], [s[0], s[1]], -2);
constraint int_lin_le ([1,-1], [s[2], s[3]], -3);
constraint int_lin_le ([1,-1], [s[1], end ], -5);
constraint int_lin_le ([1,-1], [s[3], end ], -4);
constraint int_lin_le_reif([1,-1], [s[0], s[2]], -2, b1);
constraint int_lin_le_reif([1,-1], [s[2], s[0]], -3, b2);
constraint bool_or(b1, b2, true);
constraint int_lin_le_reif([1,-1], [s[1], s[3]], -5, b3);
constraint int_lin_le_reif([1,-1], [s[3], s[1]], -4, b4);
constraint bool_or(b3, b4, true);
solve minimize end;
```
Critical Flattening Steps

- All standard in language compilers
- Constant folding
- Common Subexpression Elimination
  - two names for the same thing is deadly for CP
  - particularly for learning solvers
- Equality tracking
  - substitution/elimination of common names
model.mzn

var 0..100: b; % no. of banana cakes
var 0..100: c; % no. of chocolate cakes

constraint 250*b + 200*c <= 4000;
constraint 2*b <= 6;
constraint 75*b + 150*c <= 2000;
constraint 100*b + 150*c <= 500;
constraint 75*c <= 500;

% maximize our profit
solve maximize
400*b + 450*c;

output ["no. of banana cakes = ", show(b), ", 
no. of chocolate cakes = ", show(c), ", 
----------
"];

data.dzn

N = 15;

prettypinter

output

b = 2;
c = 2;
----------

solver
Automatic Reformulation
Sets

- **MiniZinc mantra**
  - your model runs on all solvers

- **Problem: Set variables are not supported**

- **Solution** `nosets.mzn` *(200 lines of code)*
  - translate set variables to arrays of booleans
    - crucial use of functions to avoid multiple translations
  - convert set operations to functions on arrays
  - no set variables in the final FlatZinc
Strings

- MiniZinc extended to include string variables
  - not yet released

- String solving not supported by most solvers
  - only Gecode+S

- Map strings to existing FlatZinc
  - Translate strings to arrays of integers
  - Map constraints on strings to constraints on arrays
  - Map string operations to operations on arrays
    - concatenation, reverse, length, regular, gcc, lexorder
  - Not that uncompetitive wrt to Gecode+S
Linearization

- The most important transformation
  - allows MiniZinc to run on MIP solvers
  - beware they are quite competitive on CP problems

- Linearization consists of
  - specialised linear global decompositions
    ```
    predicate alldifferent(array[int] of var int: x)
    = forall(j in array_dom(x))
       (sum(i in index_set(x))(x[i] == j) <= 1);
    ```
  - general linearization by “big M” methods
  - special treatment of constraints on variables domains (x \in S)
  - **NEW**: some globals treated as separators, e.g. circuit
MiniZinc flattens to FlatZinc

- many decisions made during flattening, e.g

```plaintext
var {2,4}: x; var {2,4}: y; var {2,4,5}: z;
constraint all_different([x,y,z]);
constraint x+y+z=12 -> y=max([x,y,z]);
```

- becomes

```plaintext
var {2,4}: x; var {2,4}: y; var {2,4,5}: z;
constraint all_different([x,y,z]);
var 2..5: i0 = max([x,y,z])
var bool: b0 = (y = i0)
var bool: b1 = (x+y+z != 12)
constraint or(b0,b1);
```
More information = better decisions

```plaintext
var {2,4}: x; var {2,4}: y; var {2,4,5}: z;
constraint all_different([x,y,z]);
var 2..5: i0 = \text{max}([x,y,z]) = 5
var bool: b0 = (y = i0) = false
var bool: b1 = (x+y+5 != 12) = true
constraint \text{or}(b0, b1);
```

finally

```plaintext
var {2,4}: x; var {2,4}: y; var {5}: z;
constraint x != y;
constraint x+y != 7;
```
Multi PassCompilation

- Multi pass compilation
- Key requirement: variable and expression paths
- Gecode first pass: Other solver second pass
- reduces model size: around 5%
- reduces run time for MIP solvers: sometimes 50%
- can improve compile time, no worse than double
Auto Tabling

- Annotate a predicate as: :: presolve(auto_table);
  
  \[
  \text{predicate rank\_apart(var 1..52: a, var 1..52: b)} = \text{table}([a,b],[| 1,2 | 1, 13 | \ldots | 52, 51 |]) \text{ in \{1522\}51 |}));
  \]

- Solution are computed
  - predicate replaced by a table constraint

- Variations
  - call-based, and instance independent

- Benefits
  - improved solving time
  - automatic reformulation of poor models

- Not done in Australia
Learning Reformulation
Symmetry Detection

- Generate symmetries of small instances
  - find which symmetries generalize across instances
- Generate candidate model symmetries
  - ask the user or use theorem proving
- Add symmetry breaking (dynamic/static) to model
- Extension to dominance
  - separate out objective and/or some constraints
  - generate symmetries
  - convert to dominance constraints

Therefore, these four symmetries form a generating set for $G$. The generating set is minimal since any proper subset is not a generating set for $G$.
 Globals Detection

- Find global constraints which are implied by the model
  - Use structure of model to find sub-problems
  - Generate candidate global constraints
  - Rank the global candidates by
    - coverage by solutions, size of global
  - Present the globals to the user in ranked order

- Was available as a web tool: minizinc.org/globalizer

- Highly important approach for non-expert modellers
  - gives a way to “lookup” the globals you need for your problems
Other Reformulations

- **Bounds versus Domain propagation**
  - we can analyse models to determine that bounds propagators will fail at the same time as domain

- **Multiple reformulations (model portfolios)**
  - e.g. map sets to multiple representations: array of bool, array of int
  - Essence tries all possible reformulations

- **Adding implied constraints**
  - similar to symmetry and globaliser: which constraints to add

- **Associative Commutative CSE**
  - use AC matching to find more CSE
  - can be much better than normal CSE on the right examples
The Holy Grail
Conclusion

- MiniZinc is a modelling language based on reformulation
  - essential to supporting varied solvers (linearization)

- Automatic Reformulation is widely used
  - language extensions by reformulation (sets, strings)
  - improving model flattening (multi-pass, auto tabling)
  - recognizing ways to improve a model (symmetry + globals detection)

- Exciting new directions
  - “Learning from Learning Solvers” CP2016 showed we can improve our models by looking at how learning solvers solve them!
  - “Automatic LBBD solving” AAAI2017 how we can create a hybrid MIP/CP solution to any model that uses the strength of both
Progress to the Holy Grail

- Better modelling languages,
  - supported by automatic reformulation
  - is a critical step towards the holy grail
- CP is closer than it was, but we need it to
  - easier to learn
  - better analysis/ transformation of models
  - faster solving
- Remember the Holy Grail is (at least in theory) unattainable
  - But that should not stop us reaching for it