Learning the Parameters of Global Constraints for Medical Scheduling

Émilie Picard-Cantin, Mathieu Bouchard, Claude-Guy Quimper, and Jason Sweeney
Émilie Picard-Cantin
## Medical Schedule

### People oriented

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
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### Shift / Task oriented

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<th>4</th>
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<tbody>
<tr>
<td>1</td>
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<td><img src="man.png" alt="Man" /></td>
<td><img src="woman.png" alt="Woman" /></td>
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**Shift**: 1, 2, 3, 4, 5, 6, 7

**Day**: 1, 2, 3, 4, 5, 6, 7

**Personnel oriented**

- **People oriented**
  - 1: Night shift
  - 2: Morning shift
  - 3: Afternoon shift

- **Shift / Task oriented**
  - 1: Night shift
  - 2: Morning shift
  - 3: Afternoon shift

---

**Orientation**

- **People oriented**
  - Day: 1, 2, 3
  - Night: 4, 5, 6, 7

- **Shift / Task oriented**
  - Morning: 1, 2, 3
  - Afternoon: 4, 5, 6
  - Night: 7
Scheduling Process

1. Expert
2. Model
3. Solver
4. Schedule
Scheduling Process

Expert → Model → Solver → Schedule
Scheduling Process

Expert → Model → Solver → Schedule
Speeding up the modelling process
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• **Challenge**: Models differ from one medical team to another
Speeding up the modelling process

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- **Observation**: There are few differences between each model.
Speeding up the modelling process

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- **Observation**: There are few differences between each model.

- **Opportunity**: For legal reasons, hospitals keep a history of their schedules.
Speeding up the modelling process

• **Challenge**: Models differ from one medical team to another

• **Observation**: There are few differences between each model.

• **Opportunity**: For legal reasons, hospitals keep a history of their schedules.

• **Goal**: To learn the models from historical data.
Recommander System

Past schedules
Recommander System

Recommander System
Past schedules

![Diagram](image-url)
Recommander System

Discovered Constraints  Recommander System  Past schedules
Recommander System

Discovered Constraints  Recommander System  Past schedules

Expert  Model  Solver  Schedule
Recommander System

Discovered Constraints -> Recommender System

Holy Grail

Past schedules

Expert -> Model

Solver -> Schedule
How to learn a constraint?

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- Do we have a limit of:
  - 2 night-shifts per week?
  - 1 night-shift every 3 days?
Which constraint was imposed?
Which constraint was imposed?

\[ x + y \leq 4 \]
Which constraint was imposed?

\[ x + y \leq 5 \]
\[ x + y \leq 4 \]
Problem Definition

• Consider a random assignment $\tilde{X}$ with $P[X_i = v] = p_v$
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- The probability of observing $\vec{X}$ is $P(\vec{X}) = \prod_{i=1}^{n} p_{X_i}$
Problem Definition

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- Consider the constraint $C([X_1, \ldots, X_n],[\alpha_1, \ldots, \alpha_m])$
Problem Definition

• Consider a random assignment \( \vec{X} \) with \( P[X_i = v] = p_v \)

• The probability of observing \( \vec{X} \) is \( P(\vec{X}) = \prod_{i=1}^{n} p_{X_i} \)

• Consider the constraint \( C([X_1, \ldots, X_n],[\alpha_1, \ldots, \alpha_m]) \)

• The probability that a random assignment satisfies \( C \) is

\[
G_C(\vec{\alpha}) = \sum_{\vec{X} \mid C(\vec{X}, \vec{\alpha})} P(\vec{X})
\]
Problem Definition

• Consider a random assignment $\tilde{X}$ with $P[X_i = v] = p_v$

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• The probability that a random assignment satisfies $C$ is

$$G_C(\vec{\alpha}) = \sum_{\vec{X} | C(\vec{X}, \vec{\alpha})} P(\vec{X})$$

• Finding $\vec{\alpha}$ consists in solving:

$$\min_{\vec{\alpha}} G_C(\vec{\alpha})$$

$$C(\vec{X}, \vec{\alpha}) \quad \forall \vec{X} \in \text{Examples}$$
How to compute $G_C(\vec{\alpha})$?

• Enumerating and summing the probability of all solutions of a constraint is slow.

• We mainly developed two techniques to compute or bound this probability

  • Using Markov chains

  • Using dynamic programming
Markov Chains

- Some constraints can naturally be encoded with an automaton.

\[ \text{SEQUENCE}([X_1, \ldots, X_n], \{1\}, 0, 2, 3) \]

1 = a night shift

at least 0

at most 2

every 3 days
Sequence Automaton

![Sequence Automaton Diagram]

- States: [ ], [0], [1], [0, 0], [0, 1], [1, 0], [1, 1], reject
- Edges:
  - [ ] → [0] with input 0
  - [0] → [0, 0] with input 0, [0] → [1] with input 1
  - [0, 0] → [0, 1] with input 0, [0, 0] → [1, 0] with input 1
  - [0, 1] → [1, 1] with input 0, [0, 1] → [1, 0] with input 1
  - [1, 0] → [1, 1] with input 0
  - [1, 1] → [0, 0] with input 1
  - [1, 1] → reject with input 0, 1
Sequence Markov Chain
Computing $G_{\text{Sequence}}(\vec{\alpha})$

- Let $M_{\vec{\alpha}}$ be the transition matrix of the Markov chain for the constraint with parameters $\vec{\alpha}$.

- One can compute the probability of reaching the reject state after reading $n$ characters by computing $M_{\vec{\alpha}}^n$.

- For every combination of $\vec{\alpha}$, compute $M_{\vec{\alpha}}^n$ and evaluate $G_C(\vec{\alpha})$.

- Keep $\vec{\alpha}$ that minimizes $G_C(\vec{\alpha})$. 
When parameters are sets

- If the parameter contains a set, there is an exponential number of combinations to explore.

\[
\text{Among}([X_1, \ldots, X_n], l, u, \tilde{z})
\]
\[
\text{Sequence}([X_1, \ldots, X_n], l, u, w, \tilde{z})
\]
\[
\text{SubSetFocus}([X_1, \ldots, X_n], l, m, \tilde{z})
\]
Branch & Bound

\[
\min_{\tilde{\alpha}} G_C(\tilde{\alpha})
\]

\[
C(\tilde{X}, \tilde{\alpha}) \quad \forall \tilde{X} \in \text{Examples}
\]

- We use this strategy to compute a bound on \( G_C(\tilde{\alpha}) \)

\[
p = \sum_v z_v \cdot p_v
\]

\[
G_C(\tilde{\alpha}) \geq G_C(\text{ext}(\tilde{\alpha}))
\]

\[
\geq \sum_{k=0}^{n} D[k, \text{ext}(\tilde{\alpha})] \left( \min_p p^k (1 - p)^{n-k} \right)
\]
Branch & Bound

\[
\min_{\bar{\alpha}} G_C(\bar{\alpha})
\]

\[
\forall \vec{X} \in \text{Examples}
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\]

Probability that a value belongs to the set.
Branch & Bound

\[
\min_{\tilde{\alpha}} G_C(\tilde{\alpha})
\]

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\]

\[
C(\bar{X}, \bar{\alpha})
\]

Examples

• We use this strategy to compute a bound on \( G_C(\bar{\alpha}) \)

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\]

Set parameters to extreme values (requires monotonicity)
Branch & Bound

\[
\min_{\tilde{\alpha}} G_C(\tilde{\alpha})
\]

\[
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Count how many solutions exist with exactly k values in the set
Branch & Bound

\[
\min_{\alpha} G_{C}(\bar{\alpha})
\]

\[
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\]

Count how many solutions exist with exactly \( k \) values in the set

Probability of a random assignment with \( k \) values in the set
Branch & Bound

\[
\min_{\bar{\alpha}} G_C(\bar{\alpha})
\]

\[
C(\bar{X}, \bar{\alpha}) \quad \forall \bar{X} \in \text{Examples}
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- We use this strategy to compute a bound on \(G_C(\bar{\alpha})\)

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\]
Studied Constraints

SubSetFocus
Sequence
Among
GCC
AtMostNValue
AtLeastNValue
AtMostBalance
AtLeastBalance
## Experiments

<table>
<thead>
<tr>
<th>Num. of examples</th>
<th>Rank of initial constraint</th>
<th>Num. of instances</th>
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Table 1: Results for SubsetFocus. Number of instances for which the initial constraint was ranked first, second, third or was not found.
Conclusion

• We were able to make a recommender system that helps experts to determine the parameters of certain constraints.
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• The system is not used!
Conclusion

• We were able to make a recommender system that helps experts to determine the parameters of certain constraints.

• The system is not used!

• It could have saved hundreds of hours in expert time.